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Pseudo-Dirac solar neutrinos

Yosef Nir

School of Natural Science, Institute for Advanced Study
Princeton, NJ 08540, USA[*] *and*
Department of Particle Physics, Weizmann Institute of Science
Rehovot 76100, Israel
E-mail: `ftnir@wicc.weizmann.ac.il`, `nir@ias.edu`

Abstract: Three of the viable solutions of the solar neutrino problem are consistent with close to maximal leptonic mixing: $\sin^2\theta_{12} = \frac{1}{2}(1-\epsilon_{12})$ with $|\epsilon_{12}| \ll 1$. Flavor models can naturally explain close to maximal mixing if approximate horizontal symmetries force a pseudo-Dirac structure on the neutrino mass matrix. An experimental determination of $|\epsilon_{12}|$ and $\mathrm{sign}(\epsilon_{12})$ can constrain the structure of the lepton mass matrices and consequently provide stringent tests of such flavor models. If both $|\epsilon_{12}|$ and Δm^2_{21} are known, it may be possible to estimate the mass scale of the pseudo-Dirac neutrinos. Radiative corrections to close to maximal mixing are negligible. Subtleties related to the kinetic terms in Froggatt-Nielsen models are clarified.

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1. Introduction

Three of the solutions of the solar neutrino problem require a large mixing angle [1, 2]:

$$\begin{aligned}
\text{LMA}: &\quad \sin^2 2\theta_{12} \sim 0.7 - 1\,, &\quad \Delta m^2_{21} &\sim (1-20)\times 10^{-5}\,\text{eV}^2\,,\\
\text{LOW}: &\quad \sin^2 2\theta_{12} \sim 0.8 - 1\,, &\quad \Delta m^2_{21} &\sim (3-30)\times 10^{-8}\,\text{eV}^2\,,\\
\text{VAC}_\text{L}: &\quad \sin^2 2\theta_{12} \sim 0.7 - 1\,, &\quad \Delta m^2_{21} &\sim (4-10)\times 10^{-10}\,\text{eV}^2\,.
\end{aligned} \tag{1.1}$$

Here LMA and LOW refer to matter-enhanced oscillations with a large mixing angle in the high and low Δm^2 ranges, respectively, while VAC_L refers to vacuum oscillations with relatively large Δm^2. The range for the mixing angle in (1.1) is close to maximal mixing, $\sin^2 2\theta_{12} = 1$. This case is particularly interesting from the theoretical point of view. It follows from a simple structure of the relevant 2×2 block in the neutrino mass matrix in the basis where the charged lepton mass matrix is diagonal:

$$M_\nu^{(2)} = m \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \tag{1.2}$$

Such a structure is easily obtained in models of horizontal symmetries [3, 4] that try to explain the observed smallness and hierarchy in the charged fermion parameters (mass ratios and mixing angles). For example, if the lepton doublets of the first two generations carry an opposite charge under a U(1) symmetry (and the relevant scalar field is neutral), then $M_\nu^{(2)}$ has the structure (1.2) in the symmetry limit.

Any horizontal symmetry must be broken in nature. An unbroken horizontal symmetry leads to either degeneracy between fermions of different generations or vanishing mixing angles (see e.g. [5] and references therein). In particular, the mass degeneracy implied by (1.2) must be broken to satisfy (1.1). The horizontal symmetry still has observable consequences if the breaking parameters are small. Then the low energy effective theory is subject to selection rules that are manifested in the smallness and hierarchy of the flavor parameters. In the case of close-to-maximal mixing, the small breaking leads to a small splitting between the masses of the two neutrinos and to a small deviation from maximal mixing, that is, the two Majorana neutrinos form a *pseudo-Dirac* neutrino:

$$\frac{\Delta m_{21}^2}{m^2} \ll 1 \,, \qquad 1-\sin^2 2\theta_{12} \ll 1 \,. \tag{1.3}$$

Here m denotes the average of m_1 and m_2. A measurement of these small effects will provide further information about the pattern of symmetry breaking and guide us in the process of selecting among the many presently viable models of horizontal symmetries. (For interesting studies of the implications of solar neutrino measurements for small entries in the neutrino mass matrix, see refs. [6, 7].)

There are three light active neutrinos in nature. (In this work we assume that these are the only light neutrinos and do not consider the possibility of light sterile neutrinos. Note that the large angle solutions of the solar neutrino problem are inconsistent with a pseudo-Dirac $\nu_e - \nu_s$ combination, such as in the model of ref. [8].) In the two generation framework, where there is a single mixing angle θ, maximal mixing is defined by maximal oscillation depth *in vacuum* and corresponds to $\sin^2 2\theta = 1$. In the three generation framework, what we mean by maximal mixing is that the disappearance probability is equivalent to that for maximal two neutrino mixing at the relevant mass scale [9]. The disappearance probability for ν_e in vacuum, $P_{e\not{e}}$, is given by

$$P_{e\not{e}} = 4|V_{e1}|^2|V_{e2}|^2 \sin^2 \Delta_{\rm sun} - 4|V_{e3}|^2 \sin^2 \Delta_{\rm atm} \,. \tag{1.4}$$

Here V_{ij} are the elements of the Maki-Nakagawa-Sakata (MNS) matrix [10] and we use the definition $\Delta_{jk} \equiv \Delta m_{jk}^2 L/4E$, and the following input from solar and atmospheric neutrino experiments:

$$|\Delta_{\rm sun}| = |\Delta_{21}| \ll |\Delta_{31}| \simeq |\Delta_{32}| = |\Delta_{\rm atm}| \,. \tag{1.5}$$

At the L/E-scale that is relevant to solar neutrinos, the second term in (1.4) averages out to $-2|V_{e3}|^2$. The only oscillatory term is the first one, and our definition of maximal mixing corresponds to

$$4|V_{e1}|^2|V_{e2}|^2 = 1\,, \tag{1.6}$$

which leads to

$$|V_{e1}|^2 = |V_{e2}|^2 = \frac{1}{2}\,, \qquad |V_{e3}|^2 = 0\,. \tag{1.7}$$

In the standard parametrization of the $V_{\rm MNS}$ matrix,

$$V_{\rm MNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{1.8}$$

where $s_{ij} \equiv \sin\theta_{ij}$ and $c_{ij} \equiv \cos\theta_{ij}$, the conditions (1.7) translate into

$$s_{12}^2 = \frac{1}{2}\,, \qquad s_{13} = 0\,. \tag{1.9}$$

By *close-to-maximal-mixing* we refer to a situation close to (1.7) or, equivalently, to (1.9):

$$\epsilon_{12} \equiv 1 - 2s_{12}^2 \ll 1\,, \qquad s_{13} \ll 1\,. \tag{1.10}$$

Our convention here is that $\Delta m_{21}^2 \equiv m_2^2 - m_1^2 > 0$, so that $\epsilon_{12} > 0$ ($\epsilon_{12} < 0$) corresponds to a situation where the lighter (heavier) state has a larger component of ν_e.

Solar neutrino experiments (and, more generally, any oscillation experiments) are sensitive to the mass-squared difference Δm_{12}^2 but not to the masses themselves. On the other hand, they can be sensitive to small deviations from maximal mixing [11, 12]. Moreover, matter oscillations (but not vacuum oscillations) are affected differently by $\epsilon_{12} > 0$ and by $\epsilon_{12} < 0$, that is, they are sensitive not only to $\sin^2 2\theta_{12}$ but also to $\sin^2\theta_{12}$. In other words, if the solar neutrino problem is solved by one of the large angle solutions, then experiments may provide us with a measurement of the sign and the size of the small parameter ϵ_{12}. The purpose of this work is to understand the potential lessons for flavor model building from solar neutrino measurements of ϵ_{12}.

Our interest lies in models where ϵ_{12}, s_{13} and $\Delta m_{21}^2/m^2$ are naturally small. We focus on models where there are no exact relations between entries of the lepton mass matrices (beyond the symmetric structure of the neutrino Majorana mass matrix). The smallness of physical parameters must then be related to the smallness of various entries in the mass matrices and not to fine tuned cancellations between various contributions. As a concerete example of such a framework we think of models of approximate abelian horizontal symmetries, but most of our results have more general applicability.

Horizontal symmetries constrain the structure of the mass matrices M_ν and M_ℓ. In section 2 we derive the dependence of the mixing angles and of $\Delta m_{12}^2/m^2$ on the entries of the lepton mass matrices. Usually, the constraints of the horizontal symmetries apply at a high energy scale. The effects of renormalization group evolution (RGE) are analyzed in section 3. For specific high energy theories of flavor, such as the Froggatt-Nielsen mechanism [13], the kinetic terms are corrected in a flavor dependent way when heavy degrees of freedom are integrated out. The effects of non-canonical kinetic terms are studied in section 4. The analysis in sections 2–4 is carried out under the simplifying assumption of CP symmetry. Effects of phases are studied within a two generation model in section 5. We apply our results to various models of abelian flavor symmetries in section 6. We summarize our conclusions in section 7.

2. From interaction basis parameters to physical parameters

Flavor models and, in particular, models with horizontal symmetries, constrain the entries of the lepton mass matrices in the interaction basis. To understand the implications of experimental constraints, one needs to express the physical observables (masses and mixing angles) in terms of the interaction basis parameters.

Given the charged lepton mass matrix M_ℓ and the neutrino mass matrix M_ν in some interaction basis,

$$-\mathcal{L}_M = (\,\overline{e_L} \quad \overline{\mu_L} \quad \overline{\tau_L}\,)\, M_\ell \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} + (\,\nu_e^T \quad \nu_\mu^T \quad \nu_\tau^T\,)\, M_\nu \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} + \text{h.c.}\,, \tag{2.1}$$

$V_{\rm MNS}$ can be found from the diagonalizing matrices V_ℓ and V_ν:

$$V_{\rm MNS} = P_\ell V_\ell V_\nu^\dagger\,, \tag{2.2}$$

where P_ℓ is a diagonal phase matrix. The unitary matrices $V_{\ell L}$ and V_ν are found from

$$V_{\ell L} M_\ell M_\ell^\dagger V_{\ell L}^\dagger = \text{diag}(m_e^2, m_\mu^2, m_\tau^2)\,, \qquad V_\nu M_\nu^\dagger M_\nu V_\nu^\dagger = \text{diag}(m_1^2, m_2^2, m_3^2)\,. \tag{2.3}$$

Our first step is to express the physical mixing angles in terms of the parameters of the diagonalizing matrices. For simplicity, we ignore CP violation, so that the mass matrices and, consequently, the diagonalizing matrices are real. (We comment on the effects of CP violating phases in section 5.) Let us define the three unitary matrices

$$\begin{aligned} R_{12}(\theta_{12}) &\equiv \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \\ R_{13}(\theta_{13}) &\equiv \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix}, \\ R_{23}(\theta_{23}) &\equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}. \end{aligned} \tag{2.4}$$

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Then, eq. (1.8) (with δ set to zero) can be rewritten as

$$V_{\rm MNS} = R_{23}(\theta_{23}) R_{13}(\theta_{13}) R_{12}(\theta_{12})\,. \tag{2.5}$$

We further parametrize the diagonalizing matrices as follows:

$$\begin{aligned} V_\nu^\dagger &= R_{23}(\theta_{23}^\nu) R_{13}(\theta_{13}^\nu) R_{12}(\theta_{12}^\nu)\,, \\ V_\ell &= R_{23}(-\theta_{23}^\ell) R_{13}(-\theta_{13}^\ell) R_{12}(-\theta_{12}^\ell)\,. \end{aligned} \tag{2.6}$$

We limit ourselves to the large class of models where there are no *exact* relations between the entries in M_ν (up to the fact that it is symmetric, that is, $(M_\nu)_{ij} = (M_\nu)_{ji}$) and in M_ℓ. Then the smallness of ϵ_{12} and s_{13} requires that the following parameters are small:

$$s_{12}^\ell,\, s_{13}^\ell,\, \epsilon_{12}^\nu,\, s_{13}^\nu \ll 1\,, \tag{2.7}$$

where

$$\epsilon_{12}^\nu \equiv 1 - 2(s_{12}^\nu)^2. \tag{2.8}$$

Evaluating to first order in the small parameters of (2.7), we obtain:

$$\begin{aligned} \epsilon_{12} &= \epsilon_{12}^\nu + 2c_{23}^\nu s_{12}^\ell - 2s_{23}^\nu s_{13}^\ell\,, \\ s_{13} &= s_{13}^\nu - s_{23}^\nu s_{12}^\ell - c_{23}^\nu s_{13}^\ell\,. \end{aligned} \tag{2.9}$$

We caution the reader that the sign of the terms that depend on s_{12}^ℓ and s_{13}^ℓ is ambiguous. In particular, we approximated $\sin 2\theta_{12}^\nu = 1$, but with the parametrization (2.6) it could equal -1. A full treatment of the sign and phase dependence of the s_{12}^ℓ contribution to ϵ_{12} is given in section 5.

Our next step is to express the parameters of the diagonalizing matrices in terms of the mass matrices. For the charged lepton sector, the expressions can be found in refs. [14, 15]. Typically, one finds $s_{12}^\ell \sim (M_\ell)_{12}/(M_\ell)_{22}$ and $s_{13}^\ell \sim (M_\ell)_{13}/(M_\ell)_{33}$. Here, we focus on the neutrino mass matrix with a pseudo-Dirac structure. If there are no exact relations between different entries in M_ν, then the most general structure that is consistent with $\epsilon_{12}^\nu \ll 1$ and $s_{13}^\nu \ll 1$ is

$$M_\nu = m \begin{pmatrix} y_{11} & Y_{12} & Y_{13} \\ Y_{12} & y_{22} & y_{23} \\ Y_{13} & y_{23} & Y_{33} \end{pmatrix}, \tag{2.10}$$

where

$$Y_{12} \sim 1\,, \qquad y_{ij} \ll 1\,. \tag{2.11}$$

As concerns Y_{13} and Y_{33}, there are three different options:

$$\begin{array}{lll} \text{(i)} & Y_{13} \lesssim 1\,, & Y_{33} \gg 1\,, \\ \text{(ii)} & Y_{13} \lesssim 1\,, & Y_{33} \equiv y_{33} \ll 1\,, \\ \text{(iii)} & Y_{13} \equiv y_{13} \ll 1\,, & Y_{33} \sim 1\,. \end{array} \tag{2.12}$$

(Explicit examples of models in the literature that realize these options are presented in section 7.) It is also convenient to define the matrix

$$\hat{M}_\nu = R_{13}^T(\theta_{13}^\nu) R_{23}^T(\theta_{23}^\nu) M_\nu R_{23}(\theta_{23}^\nu) R_{13}(\theta_{13}^\nu) \,. \tag{2.13}$$

By definition, it is block diagonal. The requirement that $\epsilon_{12}^\nu \ll 1$ restricts the form of the (12) block:

$$\hat{M}_\nu = m \begin{pmatrix} \delta_1 & 1 & 0 \\ 1 & \delta_2 & 0 \\ 0 & 0 & Y_3 \end{pmatrix} ; \qquad |\delta_1|, |\delta_2| \ll 1 \,. \tag{2.14}$$

Both $\Delta m_{21}^2/m^2$ and ϵ_{12}^ν depend on only δ_1 and δ_2:

$$\begin{aligned} \frac{\Delta m_{12}^2}{m^2} &= 2|\delta_1^* + \delta_2| \,, \\ \epsilon_{12}^\nu &= \frac{|\delta_2|^2 - |\delta_1|^2}{2|\delta_1^* + \delta_2|} \,. \end{aligned} \tag{2.15}$$

We now present, for the three cases of eq. (2.12), expressions for s_{23}^ν, s_{13}^ν, δ_1 and δ_2 to first order in the small parameters y_{ij}. For case (i), we take $Y_{12} = 1$ and obtain:

$$\begin{aligned} s_{13}^\nu &= \frac{Y_{13}}{Y_{33}} \,, & s_{23}^\nu &= \frac{(s_{13}^\nu Y_{12} + y_{23})}{Y_{33}} \,, \\ \delta_1 &= y_{11} - \frac{Y_{13}^2}{Y_{33}} \,, & \delta_2 &= y_{22} \,. \end{aligned} \tag{2.16}$$

For case (ii), we take $c_{23}^\nu Y_{12} - s_{23}^\nu Y_{13} = 1$ and obtain:

$$\begin{aligned} \tan\theta_{23}^\nu &= -\frac{Y_{13}}{Y_{12}} \,, & s_{13}^\nu &= c_{23}^\nu s_{23}^\nu (y_{33} - y_{22}) - ((c_{23}^\nu)^2 - (s_{23}^\nu)^2) y_{23} \,, \\ \delta_1 &= y_{11} \,, & \delta_2 &= (c_{23}^\nu)^2 y_{22} + (s_{23}^\nu)^2 y_{33} - 2 s_{23}^\nu c_{23}^\nu y_{23} \,. \end{aligned} \tag{2.17}$$

For case (iii), we take $Y_{12} = 1$ and obtain:

$$\begin{aligned} s_{23}^\nu &= \frac{y_{23} Y_{33} + y_{13} Y_{12}}{Y_{33}^2 - Y_{12}^2} \,, & s_{13}^\nu &= \frac{y_{13} Y_{33} + y_{23} Y_{12}}{Y_{33}^2 - Y_{12}^2} \,, \\ \delta_1 &= y_{11} \,, & \delta_2 &= y_{22} \,. \end{aligned} \tag{2.18}$$

We would like to emphasize several points related to the results derived above (some of the statements below were previously made in ref. [16] in the context of a specific class of textures for the Dirac and Majorana mass matrices in the seesaw model):

a) Eq. (2.15) implies that flavor models where ϵ_{12}^ν gives the dominant contribution to ϵ_{12} can be strongly constrained by a measurement of ϵ_{12}. Since δ_1 and δ_2 depend on different entries of M_ν, we expect no exact cancellations in their contribution to ϵ_{12}^ν. Consequently, one will be able to use the measured *size* of ϵ_{12} to estimate the size of the larger between $|\delta_1|$ and $|\delta_2|$, and the *sign* of ϵ_{12} to tell which is larger.

b) Eq. (2.9) implies that observable deviations from maximal mixing in vacuum oscillations, $1 - \sin^2 2\theta_{12} = \epsilon_{12}^2 \neq 0$, can strongly constrain flavor models. For models with a small s_{23}^ν, we have

$$\epsilon_{12}^2 \sim \max\left(\frac{\delta_2^2}{4}, \frac{\delta_1^2}{4}, 4(s_{12}^\ell)^2\right). \tag{2.19}$$

If vacuum oscillations show an observable deviation from maximal mixing, say, $\epsilon_{12}^2 \sim 0.1$, it would be difficult to explain it with a parametrically suppressed δ_2, δ_1 and s_{12}^ℓ. The accidental factor of sixteen, however, between the s_{12}^ℓ and the δ_i contributions in eq. (2.19) favors s_{12}^ℓ as the major source for such a large effect.

c) Eq. (2.15) reveals interesting relations between the mass hierarchy and the mixing. The parameters ϵ_{12}^ν and $\Delta m_{21}^2/m^2$ are of the same order of magnitude. Therefore, within models where ϵ_{12}^ν gives the dominant contribution to ϵ_{12}, one will be able to use the measured values of ϵ_{12} and Δm_{21}^2 to estimate the mass scale m of the pseudo-Dirac neutrino pair. If the contributions to ϵ_{12} related to s_{12}^ℓ and/or to s_{13}^ℓ are larger than the contribution related to ϵ_{12}^ν, then the relation between ϵ_{12} and $\Delta m_{21}^2/m^2$ is lost and, in particular, $\epsilon_{12} \gg \Delta m_{21}^2/m^2$ is possible. In any case, if there are no exact relations between entries of the lepton mass matrices, we expect

$$|\epsilon_{12}| \gtrsim \frac{\Delta m_{21}^2}{m^2}. \tag{2.20}$$

This relation can be used in two ways. First, measurements of $|\epsilon_{12}|$ and of Δm_{21}^2 would give a lower bound on m. Second, in our framework we have $m^2 \lesssim \Delta m_{\rm atm}^2$ and therefore we expect

$$|\epsilon_{12}| \gtrsim \frac{\Delta m_{\rm sun}^2}{\Delta m_{\rm atm}^2}. \tag{2.21}$$

This constraint is particularly powerful if the LMA solution (see eq. (1.1)) is realized in nature since then $\Delta m_{\rm sun}^2/\Delta m_{\rm atm}^2 \sim 0.01$.

3. Radiative corrections

We consider the effect of radiative corrections on neutrino mass matrices which at a high energy scale Λ have the pseudo-Dirac structure (2.10). In particular, we ask whether at some low energy scale μ that is relevant to the solar neutrinos, a significant deviation from maximal mixing could be induced by renormalization group evolution (RGE). We take as our framework the minimal supersymmetric Standard Model. (Our results apply also to the Standard Model, but there the smallness of the charged lepton Yukawa couplings guarantees that the radiative corrections are negligible for our purposes.)

The important parameter for our purposes is related to the Yuakawa coupling of the tau lepton:

$$\epsilon_\tau \equiv -\frac{g_\tau^2}{(4\pi)^2}(1+\tan^2\beta)\ln\frac{\Lambda}{\mu}\,. \tag{3.1}$$

Here $g_\tau(1+\tan^2\beta)^{1/2} = m_\tau/\langle\phi_d\rangle$ is the tau Yukawa coupling in the supersymmetric standard model. The ϵ_τ parameter could be of $\mathcal{O}(0.01)$ for large $\tan\beta$. (Within the SM, one has to replace $(1+\tan^2\beta)$ with $-1/2$, which gives $\epsilon_\tau \sim 10^{-6}$.) Define a matrix

$$I_\tau = \mathrm{diag}(1,1,1+\epsilon_\tau)\,. \tag{3.2}$$

We denote the neutrino mass scale at the high scale Λ by M_ν^{HE}. Then, up to universal corrections and negligibly small effects of the muon and electron Yukawa couplings, the renormalized neutrino mass matrix at a scale μ below Λ is given in logarithmic approximation by [17, 18]

$$M_\nu = I_\tau M_\nu^{\mathrm{HE}} I_\tau\,. \tag{3.3}$$

In this section, the parameters that relate to M_ν^{HE} and to its diagonalization are denoted, as before, by s_{ij}^ν and δ_i. They can be expressed in terms of the entries of M_ν^{HE} according to equations (2.16), (2.17) and (2.18). In other words, we have

$$M_\nu^{\mathrm{HE}} = mR_{23}(\theta_{23})R_{13}(\theta_{13})\begin{pmatrix}\delta_1 & 1 & 0\\ 1 & \delta_2 & 0\\ 0 & 0 & Y_3\end{pmatrix}R_{13}^T(\theta_{13})R_{23}^T(\theta_{23})\,. \tag{3.4}$$

The parameters that relate to M_ν and to its diagonalization will be denoted by $\hat{s}_{ij}$ and $\hat{\delta}_i$. The difference between them and the corresponding s_{ij} and δ_i parameters vanishes in the limit $\epsilon_\tau \to 0$. The main question that we would like to investigate is whether the differences $\hat{\delta}_{1,2} - \delta_{1,2}$ are of $\mathcal{O}(\epsilon_\tau)$ or $\ll \mathcal{O}(\epsilon_\tau)$. In the latter case the radiative corrections can be safely neglected.

After a cumbersome but straightforward calculation, we find the following leading corrections:

$$\begin{aligned}
\hat{s}_{23}^\nu - s_{23}^\nu &= \frac{1+Y_3^2}{1-Y_3^2}(c_{23}^\nu)^2 s_{23}^\nu\epsilon_\tau + \mathcal{O}(s_{13}^\nu\epsilon_\tau)\,,\\
\hat{s}_{13}^\nu - s_{13}^\nu &= \frac{2Y_3}{1-Y_3^2}c_{23}^\nu s_{23}^\nu\epsilon_\tau + \mathcal{O}(s_{13}^\nu\epsilon_\tau)\,,\\
\hat{\delta}_1 - \delta_1 &= \frac{4Y_3^2}{Y_3^2-1}s_{13}^\nu c_{23}^\nu s_{23}^\nu\epsilon_\tau + \mathcal{O}(\epsilon_\tau^2)\,,\\
\hat{\delta}_2 - \delta_2 &= -\frac{4}{Y_3^2-1}s_{13}^\nu c_{23}^\nu s_{23}^\nu\epsilon_\tau + \mathcal{O}(\epsilon_\tau^2)\,.
\end{aligned} \tag{3.5}$$

From the expressions for $\hat{\delta}_1$ and $\hat{\delta}_2$, we obtain:

$$\begin{aligned}
\frac{\Delta m_{21}^2}{m^2} &= 2|\delta_1+\delta_2+4s_{13}^\nu c_{23}^\nu s_{23}^\nu\epsilon_\tau|\,, \qquad (3.6)\\
\hat{\epsilon}_{12}^\nu &= \frac{(\delta_2-\delta_1)}{2}\,\frac{\delta_2+\delta_1+4s_{13}^\nu c_{23}^\nu s_{23}^\nu\epsilon_\tau}{|\delta_1+\delta_2+4s_{13}^\nu c_{23}^\nu s_{23}^\nu\epsilon_\tau|} + 2\frac{1+Y_3^2}{1-Y_3^2}\,\frac{(\delta_2+\delta_1)s_{13}^\nu c_{23}^\nu s_{23}^\nu\epsilon_\tau}{|\delta_1+\delta_2+4s_{13}^\nu c_{23}^\nu s_{23}^\nu\epsilon_\tau|}\,.
\end{aligned}$$

Eq. (3.6) should be compared to eq. (2.15).

We would like to emphasize the following points concerning equations (3.5) and (3.6):

a) The change in s^ν_{23} is small, $(\hat{s}^\nu_{23} - s^\nu_{23})/s^\nu_{23} = \mathcal{O}(\epsilon_\tau)$.

b) The difference $\hat{s}^\nu_{13} - s^\nu_{13}$ is suppressed beyond the naive estimate of ϵ_τ: for large (small) Y_3 it is further suppressed by $1/Y_3$ (Y_3) while for $Y_3 \sim 1$ it is suppressed by the small s^ν_{23}. Effectively then we have $(\hat{s}^\nu_{13} - s^\nu_{13})/s^\nu_{13} = \mathcal{O}(\epsilon_\tau)$.

c) Our main result is that the RGE-induced deviation from maximal mixing and mass splitting are suppressed by $\epsilon_\tau s_{13}$. A combination of the CHOOZ result [19] and the SuperKamiokande results on atmospheric neutrinos [20] implies that s_{13} is small [21, 22]. Consequently, the $\epsilon_\tau s_{13}$ suppression factor is constrained to be below $\mathcal{O}(10^{-3})$. In the limit $s_{13} = 0$, the leading effects are of order ϵ_τ^2 [23].

To summarize the results of this section: we find that the contribution from radiative corrections to the deviation from maximal mixing is suppressed beyond the smallness of ϵ_τ. The leading corrections to ϵ^ν_{12}, s^ν_{13} and $\Delta m^2_{21}/m^2$ are $\mathcal{O}[\epsilon_\tau \times \max(s_{13}, \epsilon_\tau)]$. Model independently, the size of the effect is not larger than $\mathcal{O}(10^{-3})$. (This correction could be important for the mass splitting in the $\mathrm{VAC_L}$ solution.) For the deviation from maximal mixing, this correction is too small to be observed.

4. Non-canonical kinetic terms

Models with horizontal symmetries predict the structure of the mass matrices in the basis where the horizontal charges are well defined. This preferred interaction basis can, in general, be different from the basis where the kinetic terms are canonically normalized [15, 24, 25]. In particular, when heavy degrees of freedom related to flavor physics are integrated out, the kinetic terms for the left-handed lepton doublets L_i ($i = 1, 2, 3$) can be modified to

$$R_{ij} L_i^\dagger \gamma^\mu \partial_\mu L_j \, , \tag{4.1}$$

where R is a hermitean matrix. By rescaling of the L_i fields we can bring the diagonal entries of R to equal unity,

$$R_{ii} = 1 \, , \quad |R_{ij}| \ll 1 \qquad (i \neq j) \, . \tag{4.2}$$

One can always find a hermitean matrix K that brings the kinetic terms back to canonical normalization [15]:

$$K^\dagger R K = \mathrm{diag}(1, 1, 1) \, . \tag{4.3}$$

If the mass matrix in the basis where the kinetic terms are of the non-canonical form (4.1) is $M_\nu^{\rm NC}$, then the true mass matrix, that is the matrix in the basis with canonical kinetic terms, is given by

$$M_\nu = K^T M_\nu^{\rm NC} K\,. \tag{4.4}$$

K has the form

$$K = \begin{pmatrix} 1 & k_{12} & k_{13} \\ k_{12}^* & 1 & k_{23} \\ k_{13}^* & k_{23}^* & 1 \end{pmatrix}. \tag{4.5}$$

For simplicity, we again neglect CP violation and take R and, consequently K, to be real.

We are interested in finding the effects of $k_{ij} \neq 0$ on the deviation from maximal mixing and on the mass splitting. Our analysis follows similar lines to our study of radiative corrections in the previous section. We take

$$M_\nu^{\rm NC} = m R_{23}(\theta_{23}^\nu) R_{13}(\theta_{13}^\nu) \begin{pmatrix} \delta_1 & 1 & 0 \\ 1 & \delta_2 & 0 \\ 0 & 0 & Y_3 \end{pmatrix} R_{13}^T(\theta_{13}^\nu) R_{23}^T(\theta_{23}^\nu). \tag{4.6}$$

The parameters that relate to the matrix M_ν of eq. (4.4) are denoted by $\hat{s}_{ij}^\nu$ and $\hat{\delta}_i$.

For the differences between $\hat{s}_{ij}^\nu$, $\hat{\delta}_i$ and the corresponding s_{ij}^ν, δ_i, we find:

$$\begin{aligned}
\hat{s}_{23}^\nu - s_{23}^\nu &= \frac{c_{23}^\nu}{Y_3^2 - 1}\left[(Y_3^2+1)k_{23}((c_{23}^\nu)^2 - (s_{23}^\nu)^2) + 2Y_3(k_{12}s_{23}^\nu + k_{13}c_{23}^\nu)\right], \\
\hat{s}_{13}^\nu - s_{13}^\nu &= \frac{1}{Y_3^2 - 1}\left[(Y_3^2+1)(k_{12}s_{23}^\nu + k_{13}c_{23}^\nu) + 2Y_3 k_{23}((c_{23}^\nu)^2 - (s_{23}^\nu)^2)\right], \\
\hat{\delta}_{2,1} - \delta_{2,1} &= 2(k_{12}c_{23}^\nu - k_{13}s_{23}^\nu)\,.
\end{aligned} \tag{4.7}$$

For the mass difference and deviation from maximal mixing, we obtain

$$\begin{aligned}
\frac{\Delta m_{21}^2}{m^2} &= 2|\delta_1 + \delta_2 + 4(k_{12}c_{23}^\nu - k_{13}s_{23}^\nu)|\,, \\
\hat{\epsilon}_{12}^\nu &= \frac{(\delta_2 - \delta_1)}{2}\,\frac{\delta_1 + \delta_2 + 4(k_{12}c_{23}^\nu - k_{13}s_{23}^\nu)}{|\delta_1 + \delta_2 + 4(k_{12}c_{23}^\nu - k_{13}s_{23}^\nu)|}\,.
\end{aligned} \tag{4.8}$$

Terms of order $s_{13}^\nu k_{ij}$ contribute with different signs to $\hat{\delta}_1$ and $\hat{\delta}_2$ and modify $\hat{\epsilon}_{12}^\nu$ in a qualitatively different way. Quantitatively, however, these effects are negligible.

Before we analyze the consequences eqs. (4.7) and (4.8), we would like to make two comments regarding the size of k_{ij} in Froggett-Nielsen type models:

1. In most models of horizontal symmetries, we have

$$k_{ij} \lesssim s_{ij}^\ell\,. \tag{4.9}$$

2. If two fields carry the same horizontal quantum numbers, $H(L_i) = H(L_j)$, we can always define these fields in such a way that $k_{ij} = 0$.

We would like to emphasize the following points:

a) The changes in s^ν_{23} and s^ν_{13} are of $\mathcal{O}(k_{ij})$. The effect can be significant for a small mixing angle. In particular, in the supersymmetric framework, if s^ν_{ij} vanishes because of holomorphy [15], we expect such zeros to be lifted by these corrections. The mixing angle is, however, still parametrically suppressed.

b) The leading effect on ϵ^ν_{12} does not change its size but, if $4(k_{12}c^\nu_{23} - k_{13}s^\nu_{23})$ is not much smaller than $\max(\delta_1, \delta_2)$, can affect its sign. (With CP violating phases, also the size of ϵ^ν_{12} is affected, but in the Froggatt-Nielsen class of models the parametric suppression remains the same.)

c) In principle, the k_{ij}-related corrections could enhance $\Delta m^2_{21}/m^2$ compared to ϵ^ν_{12} and therefore avoid (2.20). However, in models where the constraint (4.9) holds, (2.20) is valid.

We conclude that, in general, in models where the kinetic terms are normalized according to (4.1), $\mathrm{sign}(\epsilon_{12})$ does not give a useful constraint.

5. An effective two generation framework

In previous sections we took all the parameters in the lagrangian to be real. To understand some of the effects of phases, we analyze a two generation model allowing for the most general phase structure.

We parametrize the two generation mixing matrix by

$$V = \begin{pmatrix} c & se^{i\beta} \\ -s & ce^{i\beta} \end{pmatrix}, \tag{5.1}$$

where $c \equiv \cos\theta_{12}$, $s \equiv \sin\theta_{12}$ and the phase β is physical but does not play a role in oscillation experiments. We parametrize the diagonalizing matrices V_ℓ and V_ν in the following way:

$$V_\ell = \begin{pmatrix} c_\ell & s_\ell e^{i\beta_\ell} \\ -s_\ell & c_\ell e^{i\beta_\ell} \end{pmatrix}, \qquad V_\nu = \begin{pmatrix} c_\nu & s_\nu e^{i\beta_\nu} \\ -s_\nu & c_\nu e^{i\beta_\nu} \end{pmatrix}. \tag{5.2}$$

Using (2.2), we can express the size of the mixing angle in terms of the four parameters s_ν, s_ℓ, β_ν and β_ℓ:

$$s^2 = c^2_\ell s^2_\nu + s^2_\ell c^2_\nu - 2\,\mathrm{Re}(c_\ell s_\ell c_\nu s_\nu e^{i(\beta_\ell - \beta_\nu)}). \tag{5.3}$$

The charged lepton mass matrix can be written as

$$M_\ell = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}. \tag{5.4}$$

Our assumption that s_ℓ is small requires that a certain combination of entries is small:

$$|s_\ell| \simeq |\delta_\ell| \ll 1\,, \tag{5.5}$$

where

$$\delta_\ell \equiv \frac{m_{11}m_{21}^* + m_{12}m_{22}^*}{|m_{21}|^2 + |m_{22}|^2 - |m_{12}|^2 - |m_{11}|^2}\,. \tag{5.6}$$

The neutrino mass matrix is given by

$$M_\nu = m \begin{pmatrix} \delta_1 & 1 \\ 1 & \delta_2 \end{pmatrix}, \qquad |\delta_i| \ll 1\,. \tag{5.7}$$

We include the effects of radiative corrections, parametrized by

$$I_\mu = \mathrm{diag}(1, 1+\epsilon_\mu)\,, \qquad \epsilon_\mu \equiv -\frac{g_\mu^2}{(4\pi)^2}(1+\tan^2\beta)\ln\frac{\Lambda}{\mu}\,, \tag{5.8}$$

and of non-canonical kinetic terms, parametrized by

$$K = \begin{pmatrix} 1 & k \\ k^* & 1 \end{pmatrix}. \tag{5.9}$$

We find:

$$\begin{aligned} \hat{\delta}_1 &= \delta_1 + 2k^*\,, \\ \hat{\delta}_2 &= \delta_2 + 2k\,. \end{aligned} \tag{5.10}$$

We can now express the mass splitting $\Delta m^2/m^2$ and the deviation from maximal mixing ϵ_{12} in terms of the three parameters δ_ℓ of eq. (5.6) and $\hat{\delta}_1$ and $\hat{\delta}_2$ of eq. (5.10):

$$\begin{aligned} \frac{\Delta m^2}{m^2} &= 2|\hat{\delta}_1^* + \hat{\delta}_2|\,, \\ \epsilon_{12} &= \frac{|\hat{\delta}_2|^2 - |\hat{\delta}_1|^2}{2|\hat{\delta}_1^* + \hat{\delta}_2|} + 2\,\mathrm{Re}\left[\delta_\ell\left(\frac{|\hat{\delta}_1^* + \hat{\delta}_2|}{\hat{\delta}_1^* + \hat{\delta}_2}\right)\right]. \end{aligned} \tag{5.11}$$

Eq. (5.11) allows us to make (or to re-emphasize) the following points:

1. We again observe the accidental factor of four between the δ_ℓ contribution and the $\hat{\delta}_i$ contribution to ϵ_{12}. A large measured value of ϵ_{12} might be a hint then to the size of δ_ℓ.

2. The usefulness of an experimental determination of $\mathrm{sign}(\epsilon_{12})$ depends on the relative size of the small parameters. If $|\delta_1|, |\delta_2| \gg |\delta_\ell|, |k|$, then $\mathrm{sign}(\epsilon_{12})$ depends on the relative *size* of $|\delta_1|$ and $|\delta_2|$ which is predicted by the models and a useful constraint can be derived. On the other hand, if $|\delta_\ell|$ and/or $|k|$ are not smaller than both $|\delta_1|$ and $|\delta_2|$, then $\mathrm{sign}(\epsilon_{12})$ depends on the relative *phases* between δ_ℓ or k and $(\delta_1^* + \delta_2)$. Since generic models of approximate horizontal symmetries do not predict the phases, we cannot derive any useful constraint.

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6. Abelian horizontal symmetries

The most natural application of our results is in the framework of approximate abelian horizontal symmetries. To understand the principles of this framework, let us take the simplest example of a horizontal symmetry, $H = \mathrm{U}(1)$, that is broken by a single small parameter. We denote the breaking parameter by λ and assign to it a horizontal charge -1. Wherever numerical values are relevant, we take $\lambda = 0.2$ (so that it is of the order of the Cabibbo angle). Within a supersymmetric framework, the following selection rules apply:

a) Terms in the superpotential that carry an integer H-charge $n \geq 0$ are suppressed by λ^n. Terms with $n < 0$ vanish by holomorphy.

b) Terms in the Kähler potential that carry an integer H-charge n are suppressed by $\lambda^{|n|}$.

We are particularly interested in the leptonic Yukawa terms:

$$-\mathcal{L}_Y = Y^{\ell}_{ij} L_i \bar{\ell}_j \phi_d + \frac{Y^{\nu}_{ij}}{M} L_i L_j \phi_u \phi_u + \text{h.c.}\,, \tag{6.1}$$

where $i = 1, 2, 3$ is a generation index, L_i are lepton doublet fields, $\bar{\ell}_j$ are lepton charged singlet fields, and ϕ_u and ϕ_d are the two Higgs fields. The couplings Y_{ij} are dimensionless Yukawa couplings and M is a high energy scale. The Yukawa terms come from the superpotential. If the sum of the horizontal charges in a particular term is a positive integer, then the resulting mass term is suppressed as follows:

$$\begin{aligned}(M_\ell)_{ij} &\sim \langle \phi_d \rangle \lambda^{H(L_i)+H(\bar{\ell}_j)+H(\phi_d)}\,,\\ (M_\nu)_{ij} &\sim \frac{\langle \phi_u \rangle^2}{M} \lambda^{H(L_i)+H(L_j)+2H(\phi_u)}\,.\end{aligned} \tag{6.2}$$

Otherwise, i.e. if the sum of charges is negative or non-integer, the Yukawa coupling vanishes. We use the $\sim$ sign to emphasize that there is an unknown, independent, order one coefficient for each term (except for the relation $(M_\nu)_{ij} = (M_\nu)_{ji}$).

To understand the possible implications of close-to-maximal mixing on theoretical model building, we imagine that future measurements will give

$$\epsilon_{12} \sim \lambda\,. \tag{6.3}$$

We examine the consequences of such a constraint on three classes of models in the literature. We find that two classes of models will be excluded, while in the other a unique model is singled out that is consistent with all the requirements.

6.1 Holomorphic zeros

Option (i) of eq. (2.12) has been realized in the framework of supersymmetric abelian horizontal symmetries, where holomorphic zeros can induce a large 23 mixing together with large 23 mass hierarchy [26]. The horizontal symmetry is $\mathrm{U}(1)_1 \times \mathrm{U}(1)_2$ with breaking parameters

$$\lambda_1(-1,0)\,, \qquad \lambda_2(0,-1)\,, \qquad \lambda_1 \sim \lambda_2 \sim \lambda = 0.2\,. \tag{6.4}$$

We impose four requirements on the model: large 23 mixing, $s_{23} \sim 1$; large hierarchy, $m_2/m_3 \ll 1$; ν_1 and ν_2 form a pseudo-Dirac neutrino, $\Delta m^2_{12} \ll m^2$; A deviation from maximal mixing given by $\epsilon_{12} \sim \lambda$ (this is the *hypothetical* constraint from solar neutrino measurements). We find that there is a single set of horizontal charge assignments to the Higgs and lepton doublets that is consistent with all four requirements:

$$\phi_u(0,0)\,, \qquad \phi_d(0,0)\,, \qquad L_1(1,0)\,, \qquad L_2(-1,1)\,, \qquad L_3(0,0)\,. \tag{6.5}$$

(The choice is single up to trivial shifts by hypercharge which is an exact symmetry of the model, by a Peccei-Quinn symmetry that is an accidental symmetry of the Yukawa sector, and by lepton number if it only changes the overall neutrino mass scale and can be absorbed in the parameter M, and up to trivial exchange of $\mathrm{U}(1)_1 \leftrightarrow \mathrm{U}(1)_2$.) We find then a unique structure for M_ν:

$$M_\nu \sim \frac{\langle\phi_u\rangle^2}{M}\begin{pmatrix} \lambda^2 & \lambda & \lambda \\ \lambda & 0 & 0 \\ \lambda & 0 & 1 \end{pmatrix}. \tag{6.6}$$

This matrix is of the form (2.10) with option (i) of eq. (2.12). Therefore, eqs. (2.16) can be applied. To have $s_{23} \sim 1$ and large enough ϵ_{12}, together with acceptable charged lepton mass hierarchy, we can choose, for example,

$$\bar\ell_1(3,4)\,, \qquad \bar\ell_2(3,2)\,, \qquad \bar\ell_3(3,0)\,, \tag{6.7}$$

which gives

$$M_\ell \sim \langle\phi_d\rangle\begin{pmatrix} \lambda^8 & \lambda^6 & \lambda^4 \\ \lambda^7 & \lambda^5 & \lambda^3 \\ \lambda^7 & \lambda^5 & \lambda^3 \end{pmatrix}. \tag{6.8}$$

The parametric suppression of the physical parameters is then as follows:

$$\frac{m_\tau}{\langle\phi_d\rangle} \sim \lambda^3\,, \qquad \frac{m_\mu}{m_\tau} \sim \lambda^2\,, \qquad \frac{m_e}{m_\mu} \sim \lambda^3\,, \tag{6.9}$$

$$\frac{\Delta m^2_{21}}{\Delta m^2_{23}} \sim \lambda^3\,, \qquad \frac{\Delta m^2_{12}}{m^2} \sim \lambda\,, \tag{6.10}$$

$$s_{23} \sim 1\,, \qquad s_{13} \sim \lambda\,, \qquad \epsilon_{12} \sim \lambda\,. \tag{6.11}$$

The corrections due to a non-canonical kinetic terms,

$$k_{23} \sim \lambda^2\,, \qquad k_{12} \sim \lambda^3\,, \qquad k_{13} \sim \lambda\,, \tag{6.12}$$

leave eqs. (6.10) and (6.11) unchanged.

Within the framework of abelian horizontal symmetries, it is particularly interesting to find predictions for relations among the physical parameters that are independent of a specific choice of horizontal charges. In the quark sector, there is a single such relation [15], $|V_{us}| \sim |V_{ub}/V_{cb}|$. In the lepton sector, when singlet neutrinos play no role, there are three such relations [27]. For the class of models where holomorphic zeros give a pseudo-Dirac structure in the 12 sector but do not affect the parameters that are related to the third generation (the model presented in this subsection belongs to this class), we have the following relations:

$$\begin{aligned} \epsilon_{12} &\sim \frac{s_{13}}{s_{23}}\,, \\ \frac{m}{m_3} &\sim s_{13}s_{23}\,, \\ \frac{\Delta m^2_{12}}{m^2} &\sim \frac{s_{13}}{s_{23}}\,. \end{aligned} \tag{6.13}$$

The first of these relations, which involves only mixing angles, can be tested if oscillation experiments measure ϵ_{12} and s_{13}. The last two relations can be combined to give another testable relation.

$$\frac{\Delta m^2_{\rm sun}}{\Delta m^2_{\rm atm}} \sim s_{13}^3 s_{23}\,. \tag{6.14}$$

6.2 $L_e - L_\mu - L_\tau$ symmetry

Option (ii) of eq. (2.12) can be realized in a particulary interesting frameowork of approximate $L_e - L_\mu - L_\tau$ symmetry [28, 29]. The symmetry is broken by small parameters, ε_+ and ε_- of charges $+2$ and -2, respectively [28]. The neutrino mass matrix has the following form:

$$M_\nu \sim \frac{\langle\phi_u\rangle^2}{M}\begin{pmatrix} \varepsilon_- & 1 & 1 \\ 1 & \varepsilon_+ & \varepsilon_+ \\ 1 & \varepsilon_+ & \varepsilon_+ \end{pmatrix}. \tag{6.15}$$

This matrix is of the form (2.10) with option (ii) of eq. (2.12). Therefore, eqs. (2.17) can be applied. We find:

$$m_{1,2} = m\,(1 \pm \mathcal{O}[\max(\varepsilon_+,\varepsilon_-)])\,, \qquad m_3 = m\mathcal{O}(\varepsilon_+)\,, \tag{6.16}$$

$$s^\nu_{23} = \mathcal{O}(1)\,, \qquad s^\nu_{13} = \mathcal{O}(\varepsilon_+)\,, \qquad \epsilon^\nu_{12} = \mathcal{O}[\max(\varepsilon_+,\varepsilon_-)]\,. \tag{6.17}$$

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The charged lepton mass matrix has the form [28]:

$$M_\ell \sim \langle \phi_d \rangle \begin{pmatrix} \lambda_e & \lambda_\mu \varepsilon_- & \lambda_\tau \varepsilon_- \\ \lambda_e \varepsilon_+ & \lambda_\mu & \lambda_\tau \\ \lambda_e \varepsilon_+ & \lambda_\mu & \lambda_\tau \end{pmatrix}, \tag{6.18}$$

where the λ_i allow for a generic approximate symmetry that acts on the SU(2)-singlet charged leptons. Such a symmetry, however, does not affect the relevant diagonalizing angles:

$$s^\ell_{23} = \mathcal{O}(1)\,, \qquad s^\ell_{13} = \mathcal{O}(\varepsilon_-)\,, \qquad s^\ell_{12} = \mathcal{O}(\varepsilon_-)\,. \tag{6.19}$$

Eqs. (6.17) and (6.19) lead to the following estimates of the physical mixing angles:

$$s_{23} = \mathcal{O}(1)\,, \qquad s_{13} = \mathcal{O}[\max(\varepsilon_+, \varepsilon_-)]\,, \qquad \epsilon_{12} = \mathcal{O}[\max(\varepsilon_+, \varepsilon_-)]\,. \tag{6.20}$$

We can also estimate the corrections due to non-canonical kinetic terms:

$$k_{23} = 0\,, \qquad k_{12}, k_{13} = \mathcal{O}[\max(\varepsilon_+, \varepsilon_-)]\,. \tag{6.21}$$

This leaves the parametric suppression of the physical parameters unchanged.

From eqs. (6.16) and (6.20) we obtain:

$$\epsilon_{12} = \mathcal{O}\left(\frac{\Delta m^2_{\rm sun}}{\Delta m^2_{\rm atm}}\right). \tag{6.22}$$

Measurements of Δm^2_{ij} and of ϵ_{12} can then lead to the exclusion of this model [28]. For example, if $\Delta m^2_{\rm sun}/\Delta m^2_{\rm atm} \leq 10^{-2}$ and $\epsilon_{12} \geq 0.1$ are established, the model will be excluded.

6.3 Models with two breaking parameters

Option (iii) of eq. (2.12), that is hierarchy of mass splittings without hierarchy of masses, has been realized in the framework of non-anomalous horizontal $\mathrm{U}(1)_H$ symmetry [30]. The symmetry is broken by two small parameters of opposite charges and equal magnitudes:

$$H(\lambda) = +1\,, \qquad H(\bar\lambda) = -1\,, \qquad \lambda = \bar\lambda \sim 0.2\,. \tag{6.23}$$

Then, the following selection rule applies: terms in the superpotential or in the Kahler potential that carry an (integer) H-charge n are suppressed by $\lambda^{|n|}$. The three neutrino masses are of the same order of magnitude, but the mass splitting between ν_1 and ν_2 is small if we have

$$\begin{aligned} &|H(L_1) + H(L_2)| = 2|H(L_3)|\,, \\ &|H(L_1) + H(L_2)| < 2|H(L_1)|, 2|H(L_2)|\,. \end{aligned} \tag{6.24}$$

From eq. (2.18) we learn that

$$\epsilon_{12}^{\nu} \sim \max\left(\lambda^{2|H(L_1)|-|H(L_1)+H(L_2)|}, \lambda^{2|H(L_2)|-|H(L_1)+H(L_2)|}\right). \tag{6.25}$$

A typical contribution to s_{12}^{ℓ} is given by

$$s_{12}^{\ell} \sim \lambda^{|H(L_1)+H(\bar{\ell}_2)|-|H(L_2)+H(\bar{\ell}_2)|}. \tag{6.26}$$

The important point here is that the first condition in eq. (6.24) requires that $H(L_1)$ and $H(L_2)$ are either both even or both odd. Eqs. (6.25) and (6.26) give then an upper bound on ϵ_{12},

$$\epsilon_{12} \lesssim \lambda^2. \tag{6.27}$$

We conclude that if experiments find $\epsilon_{12} \sim \lambda$, this type of models will be strongly disfavored.

6.4 Alignment

We would like to make a comment on a particular class of supersymmetric models, where there is no degeneracy among the sleptons and the only mechanism to suppress the supersymmetric contributions to lepton flavor changing decays is *alignment* [31, 15, 27], that is small mixing angles in the neutralino-lepton-slepton couplings. In such models, there is a strong constraint on s_{12}^{ℓ} (see e.g. [32]):

$$\frac{B(\mu \to e\gamma)}{1.2 \times 10^{-11}} \sim \left(\frac{s_{12}^{\ell}}{2 \times 10^{-3}}\right)^2 \left(\frac{100\,\mathrm{GeV}}{m(\tilde{\ell})}\right)^4 < 1, \tag{6.28}$$

where $m(\tilde{\ell})$ is the average slepton mass. In these models it is then particularly difficult to explain a large deviation from maximal mixing. If the dominant source of deviation from maximal mixing is s_{12}^{ℓ}, we have

$$\epsilon_{12} \simeq 2s_{12}^{\ell} \lesssim 4 \times 10^{-3} \left(\frac{m(\tilde{\ell})}{100\,\mathrm{GeV}}\right)^2. \tag{6.29}$$

7. Conclusions

If the solar neutrino problem is solved by a large mixing angle solution, and if the mixing is established to be close to maximal but not precisely maximal, then interesting constraints for theoretical model building would arise. Specifically, experiments may measure the size and the sign of the small parameter ϵ_{12} defined by

$$\sin^2\theta_{12} \equiv \frac{1}{2}(1 - \epsilon_{12}). \tag{7.1}$$

Flavor models can account for a small ϵ_{12} by forcing a pseudo-Dirac structure on the neutrino mass matrix through an approximate horizontal symmetry,

$$M_\nu^{(2)} \sim m \begin{pmatrix} \delta_1 & 1 \\ 1 & \delta_2 \end{pmatrix}, \qquad |\delta_1|, |\delta_2| \ll 1\,. \tag{7.2}$$

We focus on models where there are no exact relations between different entries of the lepton mass matrices (except for $(M_\nu)_{ij} = (M_\nu)_{ji}$). Our main points are the following:

1. The most powerful constraints would arise if δ_1 and/or δ_2 are the dominant sources of ϵ_{12}. Then the size of $|\epsilon_{12}|$ gives the size of the larger between $|\delta_1|$ and $|\delta_2|$ while the sign of ϵ_{12} determines which of the two is larger. Moreover, the mass scale of the solar neutrinos (and not only their mass-squared splitting) can be estimated, $m^2 \sim \Delta m^2_{21}/|\epsilon_{12}|$.

2. If the dominant source of ϵ_{12} is a small angle in the diagonalizing matrix for the charged lepton mass matrix, s^ℓ_{12}, then $|\epsilon_{12}|$ constrains the size of s^ℓ_{12} but $\text{sign}(\epsilon_{12})$ is unlikely to test the theoretical models. The order of magnitude relation between $|\epsilon_{12}|$ and $\Delta m^2_{12}/m^2$ is lost, but there is still a useful inequality, $|\epsilon_{12}| \gtrsim \Delta m^2_{21}/m^2$.

3. Radiative corrections do not play a significant role in ϵ_{12} and in s_{13}. They are supppressed by the tau Yukawa coupling, by a loop factor and by s_{13}. Consequently, their effect is below the level of 10^{-3}.

4. In models of horizontal symmetries where the kinetic terms are not canonically normalized, $\text{sign}(\epsilon_{12})$ depends on the kinetic terms as well and is unlikely to test the models.

It remains to be seen whether future developments in solar neutrino experiments would make a convincing case for the intriguing scenario of pseudo-Dirac neutrinos [33].

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